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Technical Notes

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Three-Dimensional Wings and Waveriders with Attached Shock Waves

Hamdi T. Hemdan*
King Saud University, Riyadh 11451, Saudi Arabia

Introduction

THE main drawback of waveriders utilizing the stream surfaces of other flowfields is that the shock wave will be attached to the leading edge only at the design Mach number. At the off-design point, the shock wave will separate from the leading edge, thus reducing the lifting force, and the three-dimensional flowfield will not be mathematically tractable. An alternative design approach that eliminates this drawback would be through direct study of flow past three-dimensional bodies with attached shock waves. A useful tool in this regard is Messiter's thin shock layer theory (TSLT). We will demonstrate that the theory is applicable in the attached shock case, and we will use it to find new waverider configurations. Besides, various shapes of triangular wings including plane delta wings, delta wings with conical thickness, inverted V-shape wings, and caret wings are covered. This Note is derived from a full-length paper, copies of which are available from the author.

Perturbation Theory

As a starting point, we have the following equations of the extended thin shock-layer theory[1] [Eq. (2)—given in Ref. 2—extends the theory to thickness effect] when conical coordinates $\tilde{y}$ and $\tilde{z}$ are used:

$$\frac{\partial v}{\partial \tilde{y}} + \frac{\partial w}{\partial \tilde{z}} = 0 \tag{1a}$$

$$(v - \tilde{y})\frac{\partial w}{\partial \tilde{y}} + (w - \tilde{z})\frac{\partial w}{\partial \tilde{z}} = 0 \tag{1b}$$

$$v[F(z), z] = F(\tilde{z}) + (w - \tilde{z})\, F'(\tilde{z}), \qquad -\Omega<\tilde{z}<\Omega \tag{2}$$

$$w[F(z), z] = -S'(\tilde{z}) \tag{3a}$$

$$v[F(z), z] = S(\tilde{z}) - \tilde{z}S'(\tilde{z}) - S'^2(\tilde{z}) - 1 \tag{3b}$$

where v and w are the crossflow speeds, F and S the body and shock wave, respectively, and Ω the Messiter's similarity parameter, which should be greater than 2 for the shock wave to be attached.

The confusion associated with the application of the TSLT to the attached shock case can be removed by utilizing Malmuth's[3] and Hui's[4] approach of seeking the flow past a plane delta wing as a small perturbation of some basic two-dimensional flow. Thus, we first define a conical coordinate z by $z = \tilde{z}/\Omega$ and use $y = \tilde{y}$, $v(\tilde{y}, \tilde{z}) = v(y, z)$, $w(\tilde{y}, \tilde{z}) = w(y, z)$, and $S(\tilde{z}) = S(z)$ into Eqs. (1-3); then for $\Omega \gg 1$, we introduce the following perturbation expansion into the transformed form of Eqs. (1-3)

$$v(y, z) = v_0(y, z) + \frac{1}{\Omega}v_1(y, z) + \frac{1}{\Omega^2}v_2(y, z) + \cdots \tag{4a}$$

$$w(y, z) = \frac{1}{\Omega}w_1(y, z) + \frac{1}{\Omega^2}w_2(y, z) + \frac{1}{\Omega^3}w_3(y, z) + \cdots \tag{4b}$$

$$S(z) = S_0(z) + \frac{1}{\Omega}S_1(z) + \frac{1}{\Omega^2}S_2(z) + \frac{1}{\Omega^3}S_3(z) + \cdots \tag{4c}$$

Equations (4) seek the flow past the three-dimensional triangular wing as a small perturbation from some basic two-dimensional flow. Substituting Eqs. (4) into the transformed form of Eqs. (1-3) and equating like powers of Ω, we get the following approximate equations

$$\frac{\partial v_0}{\partial y} = 0 \tag{5a}$$

$$(v_0 - y)\frac{\partial w_1}{\partial y} - z\frac{\partial w_1}{\partial z} = 0 \tag{5b}$$

$$v_0[F(z), z] = F(z) - zF'(z) \tag{5c}$$

$$v_0[S_0(z), z] = -1 + S_0(z) - zS_0'(z) \tag{5d}$$

$$w_1[S_0(z), z] = -S_0'(z) \tag{5e}$$

$$\frac{\partial v_1}{\partial y} = 0 \tag{6a}$$

$$(v_0 - y)\frac{\partial w_2}{\partial y} - z\frac{\partial w_2}{\partial z} = -v_1\frac{\partial w_1}{\partial y} \tag{6b}$$

$$v_1[F(z), z] = 0 \tag{6c}$$

$$v_1[S_0(z), z] = S_1(z) - zS_1'(z) \tag{6d}$$

$$w_2[S_0(z), z] = S_1(z) - zS_1'(z) \tag{6e}$$

$$\frac{\partial v_2}{\partial y} = -\frac{\partial w_1}{\partial z} \tag{7a}$$

$$v_2[F(z), z] = F'(z)w_1[F(z), z] \tag{7b}$$

$$v_2[S_0(z), z] = S_2(z) - zS_2'(z) - S_0'^2(z) \tag{7c}$$

Equations (5-7) give a first, second, and third approximation.

Solutions

Using a set of characteristic coordinates, the solution of Eqs. (5-7) can be found as

$$v_0(y, z) = F(z) - zF'(z), \qquad S_0(z) = 1 + c\,|z| + F(z) \tag{8a}$$

$$w_1(y, z) = -c\,\mathrm{sgn}(z) - F'(\xi), \qquad \xi = \frac{z}{y - F(z) - c\,|z|} \tag{8b}$$

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*Professor, Mathematics Department, P.O. Box 2455. Member AIAA.

$$S_1(z) = a|z|, \qquad v_1(y, z) = 0$$

$$w_2(y, z) = -a \operatorname{sgn}(\xi)[1 + \xi^2 F'(\xi)] \tag{8c}$$

$$v_2(y, z) = c \operatorname{sgn}(z)\left[F'\left(\frac{\operatorname{sgn}(z)}{-c}\right) - F'(z) - F'(\xi)\right]$$

$$-F'(\xi)F'(z) - \int_{\operatorname{sgn}(z)/-c}^{\xi} \frac{1}{\xi} F''(\xi)\,\mathrm{d}\xi \tag{8d}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{S_2}{z}\right) = [-S_0'^2 - v_2(S_0, z)]/z^2, \qquad z^2 S_2''(z) = F''(z) \tag{8e}$$

where a and c are constants. Equation (8e) shows that a regular solution should satisfy $F''(z) = z^2 g(z)$, where $g(z)$ is a regular function. We can now investigate the flowfields predicted. We start by a singular solution for which $g(z) = b_0/z^2$, b_0 being a constant. This gives $F''(z) = b_0$, which shows that it is not possible to have an attached shock solution for which the body is a parabolic arc. This same result has been obtained by the present author[5] employing a different approach. The flowfield of a regular solution will be given by

$$F(z) = b_0 + b_1|z| + \iint z^2 g(z)\,\mathrm{d}z\,\mathrm{d}z$$

$$w(y, z) = \frac{1}{\Omega}[\operatorname{sgn}(z) - F'(\xi)] \tag{9a}$$

$$S(z) = (1 - |z|)\left(1 + \frac{k_0}{\Omega^2}\right) + b_0 + b_1|z| + \frac{1}{\Omega^2}[\Phi(z) - |z|\Phi(1)]$$

$$+ \iint z^2 g(z)\,\mathrm{d}z\,\mathrm{d}z \tag{9b}$$

$$v(y, z) = F(z) - zF'(z) - \frac{1}{\Omega^2}\int_{\operatorname{sgn}(z)}^{\xi} \frac{1}{\zeta} F''(\zeta)\,\mathrm{d}\zeta + \frac{1}{\Omega^2}\operatorname{sgn}(z)$$

$$\{F'(z) + F'(\xi) - F'[\operatorname{sgn}(z)]\} - \frac{1}{\Omega^2}F'(z)F'(\xi) \tag{9c}$$

$$\Phi(z) = \iint g(z)\,\mathrm{d}z\,\mathrm{d}z \tag{9d}$$

where b_0 and b_1 are arbitrary constants, and k_0 is given by

$$k_0 = 1 - F'(1) + \int \zeta g(\zeta)\,\mathrm{d}\zeta$$

Various three-dimensional wings with triangular planform may be tried in Eqs. (9). First, a plane delta will have $F(z) = 0$ and we get $b_0 = b_1 = g(z) = v(y,z) = 0$, and

$$S(z) = \left(1 + \frac{1}{\Omega^2}\right)(1 - |z|), \qquad w(y, z) = \frac{1}{\Omega}\operatorname{sgn}(z)$$

This solution shows that the TSLT gives a V-shaped shock wave that is nondifferentiable at the central axis. The fluid speed normal to the surface is zero everywhere and the side wash has uniform magnitude and is discontinuous at the central axis. This solution appears to be the most simple solution given to this problem so far. Second, assume that $F(z)$ is differentiable with $F(1) = 0$ and $g(z) = -b_2 = \text{const}$. Thus we get

$$b_1 = 0, \qquad F(z) = b_0(1 - z^4)$$

$$w(y, z) = \frac{1}{\Omega}[\operatorname{sgn}(z) + 4b_0\xi^3] \tag{10a}$$

$$S(z) = \left(1 + \frac{k_0}{\Omega^2}\right)(1 - |z|) + b_0(1 - z^4) + \frac{6b_0}{\Omega^2}[|z| - z^2] \tag{10b}$$

$$v(y, z) = -\frac{2b_0}{\Omega^2} + b_0(1 + 3z^4)$$

$$+ \frac{12b_0}{\Omega^2}\left[\frac{1}{2}\xi^2 - \frac{4}{3}b_0(\xi z)^3 - \frac{|z|}{3}\left(z^2 + \frac{\xi^3}{z}\right)\right] \tag{10c}$$

Equations (10) give a nonuniform flow with curved shock that is nondifferentiable at $z = 0$. Also the side wash w is nondifferentiable at the same point.

Now consider inverted V wings for which $F(z) = b_0 + b_1|z|$, $g(z) = 0$. The flowfield will be given by

$$S(z) = 1 + b_0 + \frac{1}{\Omega^2}(1 - b_1) - \left[(1 - b_1)\left(1 + \frac{1}{\Omega^2}\right)\right]|z| \tag{11a}$$

$$v(y, z) = b_0 + \frac{b_1}{\Omega^2}(1 - b_1), \qquad w(y, z) = \frac{1}{\Omega}(1 - b_1)\operatorname{sgn}(z) \tag{11b}$$

Equation (11a) shows that the shock wave of an inverted V cross section consists of two planar segments and is nondifferentiable at the central axis. Also, w is discontinuous at the central axis, and the fluid speed is uniform everywhere. This simple uniform flow prediction is characteristic of the present approximation. A special case of the inverted V-wing is the caret wing for which $S''(z) \equiv 0$. Equation (11a) gives $b_1 = 1$ and we get the following uniform flow

$$F(z) = b_0 + |z|, \quad S(z) = 1 + b_0, \quad v(y, z) = \mathrm{b}_0, \quad w(y, z) = 0$$

Thus far we covered flowfields with nondifferentiable shock waves. Now we consider the more realistic case of a differentiable shock wave. A body supported by such a shock can be used as the lower surface of a three-dimensional curved waverider. Equation (9b) shows that $S(z)$will be attached and differentiable for all values of z if $b_1 = 1$ and $k_0 = -\Phi(1)$, and

$$\int (1 - z^2) g(z)\,\mathrm{d}z\bigg|_{z=1} = 0 \tag{12}$$

Various regular functions $g(z)$ could be tried into Eq. (12). First, taking $g(z) = \text{const} = -b_2$ gives the plane caret-wing solution found before. Second, take $g(z) = -b_2 + b_3 z^2$, where b_2 and b_3 are constants. Equation (12) gives $b_3 = 5b_2$ and the condition $k_0 = -\Phi(1)$ gives $k_0 = b_2/12$. Hence, the flowfield of the waverider will be

$$F(z) = b_0 + |z| - \frac{b_2}{12}(z^4 - 2z^6), \quad w(y, z) = \frac{b_2}{3\Omega}\xi^3(1 - 3\xi^2) \tag{13a}$$

$$S(z) = 1 + b_0 + \frac{b_2}{12\Omega^2} - \frac{b_2}{12}(z^4 - 2z^6) - \frac{b_2}{2\Omega^4}\left(z^2 - \frac{5}{6}z^4\right) \tag{13b}$$

$$v(y, z) = b_0 + \frac{b_2}{2}z^4\left(\frac{1}{2} - \frac{5}{3}z^2\right) + \frac{1}{\Omega^2}\left[b_2\left(\frac{1}{2}\xi^2 - \frac{5}{4}\xi^4 + \frac{3}{4}\right)\right.$$

$$\left. - \frac{2b_2}{3} - \left(\frac{b_2}{3}\right)^2 (z\xi)^3 (1 - 3z^2)(1 - 3\xi^2)\right] \tag{13c}$$

Equations (13) give a closed-form simple solution for the flow past a waverider that is curved, continuous, and differentiable at all cross-section points except $z = 0$. The shock wave is curved, continuous, and differentiable at all points. In fact, $F(z)$ appears to be a modified inverted V shape, which has some small curvature of order $\mathcal{O}(\Omega^{-2})$. Notice that v and w are continuous regular functions but they are not differentiable at $z = 0$. Thus, discontinuities in the flow occur in the derivatives but not in the functions themselves. This conclusion has been reached by the present author[5] for delta wings with conical thickness through an exact study of the TSLT. Other classes of waveriders can be studied the same way.

Finally, we calculate the surface pressure. As already noted, the pressure is decoupled from the other functions in the

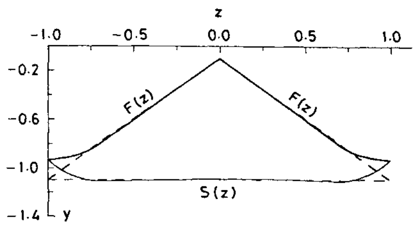

Fig. 1 Typical curved waverider (——) and a caret wing (- - -) and the supporting shock waves: $\gamma = 1.4$; $M_\infty = 10$; $\alpha = 25$ deg.; $b_0 = 0.1$; $b_2 = -2$

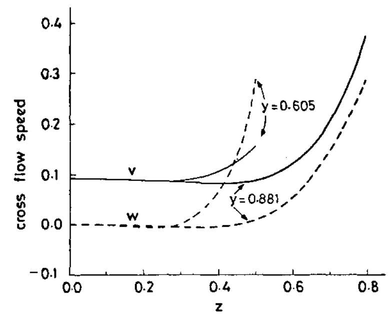

Fig. 2 Crossflow speeds for a curved waverider: $b_0 = 0.1$; $b_2 = -2$; $\gamma = 1.4$; $M_\infty = 10$; $\alpha = 25$ deg; $\Lambda = 45$ deg.

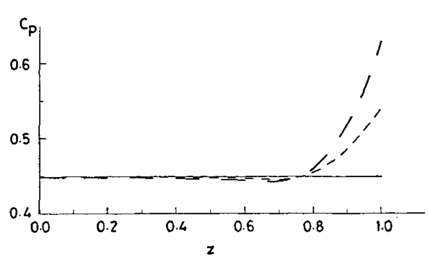

Fig. 3 Surface pressure of a caret wing and curved waveriders: $b_0 = 0.1$; $\gamma = 1.4$; $M_\infty = 10$; $\alpha = 25$ deg; $\Lambda = 45$ deg.; caret wing (——); $b_2 = -2$ (— —); $b_2 = -1$ (- - -).

TSLT and can be found by direct integration after introducing $\bar{z}$ into the pressure equations. It suffices here to give the final results as follows. First, for inverted V-shape wings, p is uniform and given by $p(y, z) = 2c_1 - 1 - c_2^2/\Omega^2$, where c_1 and c_2 are constants, and for a caret wing, it is also uniform and given by $p(y, z) = 1 + 2b_0$. The plane delta wing will have $p = 1 + 1/\Omega^2 - 1/\Omega^4[2 + 1/\Omega^2]$. The pressure has also been calculated in the cases where $F(z)$ is differentiable with $F(1) = 0$ and for curved waveriders, however, the details are omitted.

Results

Now we present some results. Figure 1 shows a caret wing and a curved waverider and their shock waves. The curved waverider shock wave has small (convex) curvature for all z. In Fig. 2, the crossflow speeds v and w on the lines $y = 0.605$ and 0.881 are shown. The figure shows that they remain almost uniform in the inboard of the flowfield but undergo some to considerable increase near the leading edges. Also noted is that the side wash w is almost zero (almost two-dimensional flow) in the inboard region. Figure 3 shows the surface pressure coefficient $C_p = (p_b - P_\infty)/\frac{1}{2}\rho_\infty U_\infty^2$, where p_b is the surface pressure and P_∞, ρ_∞, and U_∞, are the freestream pressure, density, and speed, respectively. Figure 3 shows the variation of C_p for a caret wing and two waveriders. The surface convex curvature, though small, has considerable effect on C_p near the leading edges.

References

[1]Messiter, A. F., "Lift of Slender Delta Wings According to Newtonian Theory," *AIAA Journal*, Vol. 1, No. 4, 1963, pp. 794-802.

[2]Hemdan, H. T., and Hui, W. H., "Unsteady and Steady Aerodynamic Forces of Slender Delta Wings According to Newtonian Theory," *Canadian Aeronautics and Space Journal*, Vol. 23, No. 4, 1976, pp. 238-251.

[3]Malmuth, N. D., "Three-Dimensional Perturbations on Hypersonic Wedge Flow," *AIAA Journal*, Vol. 2, No. 8, 1964, pp. 1383-1389.

[4]Hui, W. H., "Supersonic and Hypersonic Flow with Attached Shock Waves over Delta Wings," *Proceedings of the Royal Society of London, Series A*, Vol. 325, 1971, pp. 251-267.

[5]Hemdan, H. T., "Waverider Configurations According to Thin Shock-layer Theory," *Acta Astronautica*, Vol. 21, No. 8, 1990, pp. 571-582.

Direct Frequency Domain Calculation of Open Rotor Noise

Donald B. Hanson*
United Technologies Corporation,
Windsor Locks, Connecticut 06096

Introduction

TESTS of the Propeller Test Assessment (PTA) Aircraft[1] provided valuable near-field noise data on a large-scale (9-ft-diam) propfan operating at its 0.8 Mach number design condition. The nacelle was designed so that its tilt could be adjusted to three pitches for the evaluation of angular inflow effects. A recent paper[2] described a comparison of noise predictions from two frequency domain computer codes with data from the PTA experiment. Unsteady loading for input to the noise theory was computed using a time-accurate Euler method sensitive to angular inflow. One of the noise codes, identified as frequency domain, Hanson (FDH), was based on Hanson's near-field frequency domain theory.[3,4] It accounts for unsteady loading but not angular inflow. The other code, identified as frequency domain, Envia (FDE), was developed by Envia based on an unpublished theory that is sensitive to angular inflow. In Hanson's theory the tangential integration over the source volume is done analytically, leading to Bessel functions in the radiation formulas. From the description given in Ref. 2, it appears that Envia's method uses numerical integration over the tangential source coordinate. This numerical approach offers the advantage that propeller loading, geometry, and inflow angle can be represented more precisely for near-field calculations.

Theoretical waveforms and harmonic directivity patterns were compared in Ref. 2 with data from a series of microphones on a wing-mounted boom outboard of the propeller parallel to the propeller axis. Unfortunately, an error in the Envia code[5] invalidated the calculations in Ref. 2. However, predictions with the Hanson code appear to represent correct use of the axial inflow theory and are the motivation for this Note. In Fig. 1, the calculations are by Nallasamy et al. using the Hanson theory, and the data are boom microphone levels from Ref. 1 with corrections to the free field using the scattering theory of Ref. 6. (Starting with the forward microphone, the corrections were 0, 0.4, 0.7, 0.9, and 0.7 dB.) As shown in the figure, these calculations correlated well with data for two of the microphones aft of the plane of rotation but overpre-

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*Principal Research Engineer, Propulsion Analysis Group, Mail Stop 1-3-BC52, Hamilton Standard Division, One Hamilton Road. Associate Fellow AIAA.